

Bayesian Spatial Hierarchical Modeling of Maximum Temperature

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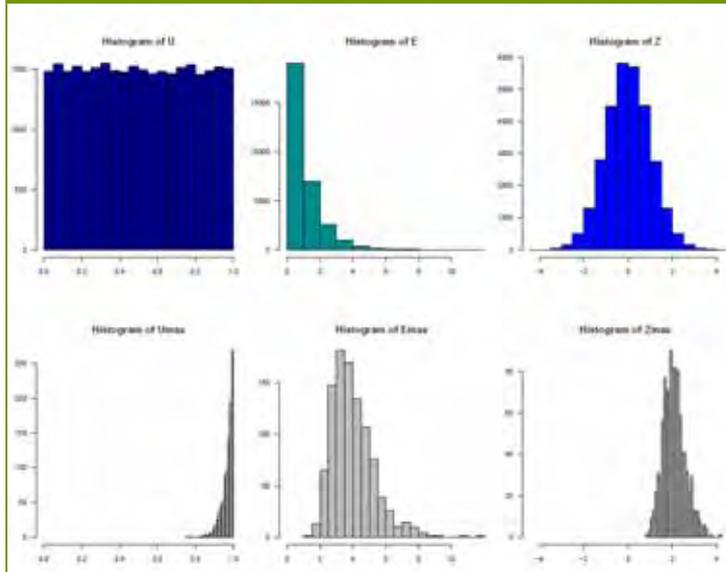
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Outline



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Introduction

Extreme Values

Hierarchical Spatial Models

Extreme Values



Data and distribution

- **Block maxima**, follows generalized extreme value (GEV) distribution;

$$G(x) = \exp \left\{ - \left(1 + \xi \frac{x - \mu}{\sigma} \right)^{-1/\xi} \right\}$$

- The three parameters (μ, σ, ξ) are called **location**, **scale** and **shape** parameters.
- **Exceedances over threshold**, another type of extremes, follow generalized Pareto distribution (GPD).

Parameter Estimation

- **Maximum Likelihood**. A common and inexpensive method in estimation parameter
- **L-moments**. Expectations of certain **linear combinations of order statistics** that work analogously to method of moments, in some cases might **outperform** the excellent MLE (Hosking (1990)).
- **Bayesian approach**. Suitable method for hierarchical model.

Hierarchical Gaussian Spatial Model



- Let $Y = (Y(\tilde{s}_1, \dots, \tilde{s}_m))^T$ are m observations of a spatial process, and define a latent spatial vector, $\eta = (\eta(s_1), \dots, \eta(s_n))^T$ where $\eta(s_i)$ is from a Gaussian spatial process and s_i is a spatial location. Observation locations $\{\tilde{s}_1, \dots, \tilde{s}_m\}$ are not necessarily coincide with $\{s_1, \dots, s_n\}$.

- A common hierarchical spatial model consist of **three stages**

$$\text{Data model} : Y|\beta, \eta, \sigma_\epsilon^2 \sim \text{Gau}(X\beta + H\eta, \sigma_\epsilon^2 I)$$

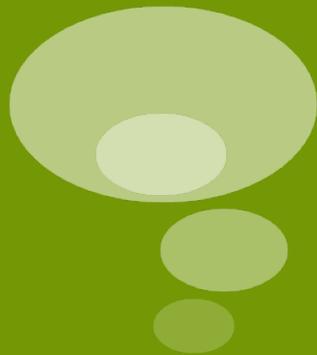
$$\text{Process model} : \eta|\theta \sim \text{Gau}(0, \Sigma(\theta))$$

$$\text{Parameter model} : [\beta, \sigma_\epsilon^2, \theta].$$

- **Bayes' rule**

$P(\text{process, parameters}|\text{data})$

$P(\text{data}|\text{process, parameters})P(\text{process}|\text{parameters}) P(\text{parameters})$



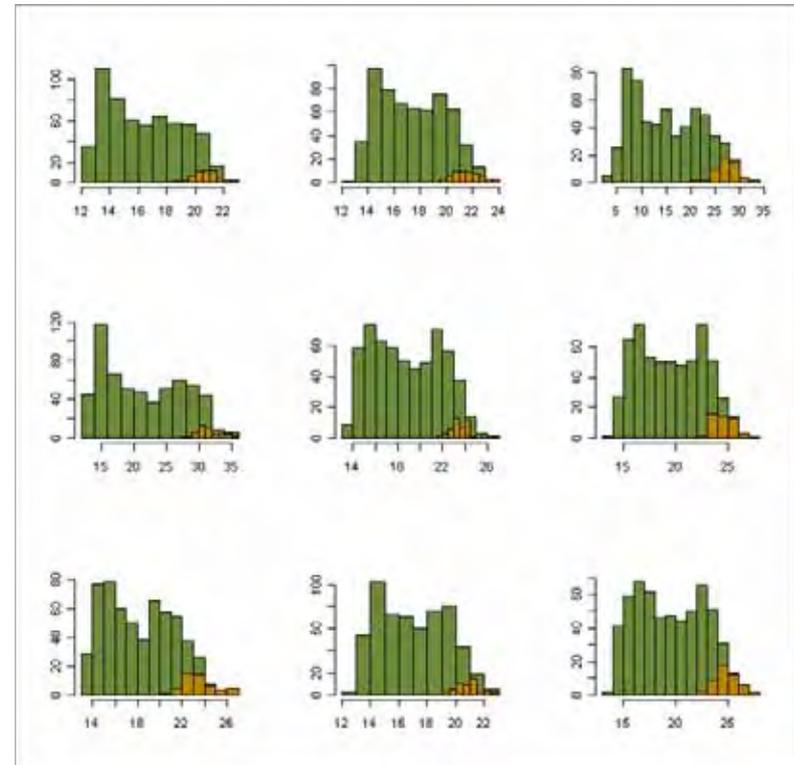
The Data

were retrieved from the
Tasmanian Partnership of
Advanced Computing portal
<http://dl.tpac.org.au/tpacportal/>

The data

- **6 set** monthly maximum temperature data, each are simulation RCM data driven by 6 different GCMs output.
- Choose **maximum** temperature for each year from 1961 to 2009. In that case, the data are simulated as if from 'control' run; no increase in CO₂.
- The spatial domain are **56 by 51**, yielding 2856 **grid cells** covering Tasmania (Latitude: 44⁰ – 39⁰S, Longitude: 143.5⁰ – 154⁰E).

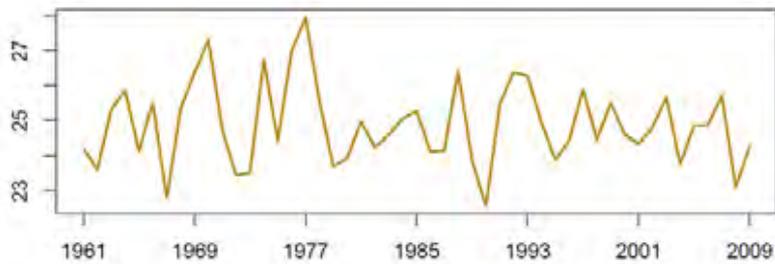
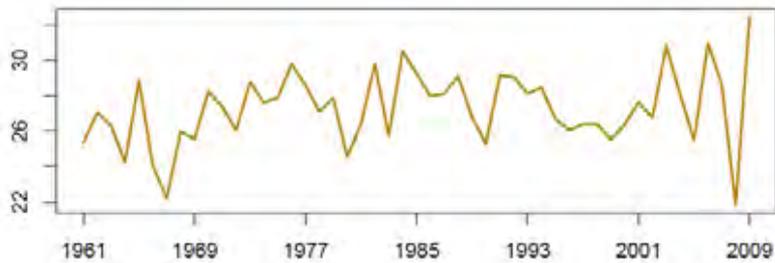
Climate Future for Tasmania used the CSIRO downscaling application CCAM to derive climate processes over Tasmania at scales of **10-15 km**.



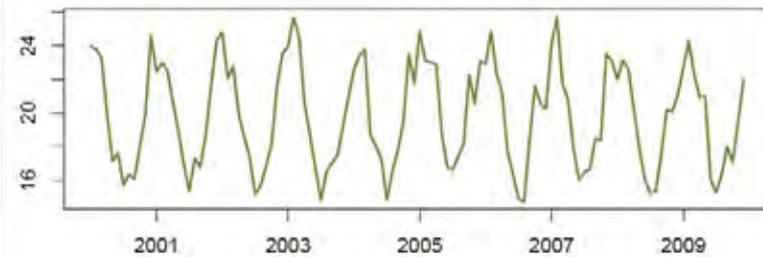
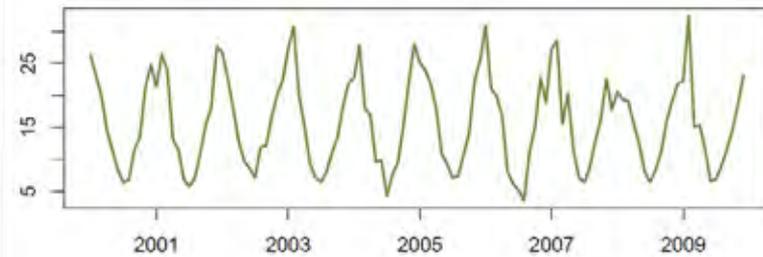
Histogram of **monthly** and **annual** maximum temperature at some randomly chosen locations

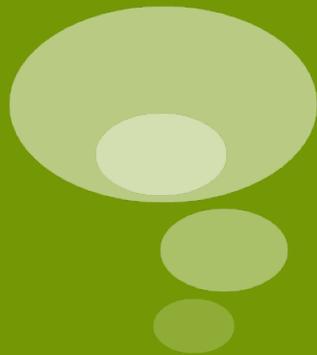


Annual time series data



Monthly time series data





The Model

follows Schliep, E. *et.al.* (2010)

model

using three stages:

data – process – prior level



Data Level

- **Assume**, $P(Z_{it} \leq z) = \exp \left\{ - \left(1 + \xi_i \frac{z - \mu_i}{\sigma_i} \right)^{-1/\xi_i} \right\}$
provided $\left(1 + \xi_i \frac{z - \mu_i}{\sigma_i} \right) > 0$ for each i and where μ_i , σ_i and ξ_i are unknown location, scale and shape parameters in grid cell i respectively.
- The **likelihood** of the data, assuming that z_{it} given μ_i , σ_i and ξ_i is conditionally independent to z_{jt} given μ_j , σ_j and ξ_j for $i \neq j$, with Martins Stedinger penalized:

$$\begin{aligned} \Pi(z_i | (\mu_i, \sigma_i, \xi_i)) &= K \prod_{i=1}^d \prod_{t=1}^{49} \exp \left\{ - \left[1 + \xi_i \left(\frac{z_{it} - \mu_i}{\sigma_i} \right) \right]^{-1/\xi_i} \right\} \\ &\times \frac{1}{\sigma_i} \left[1 + \xi_i \left(\frac{z_{it} - \mu_i}{\sigma_i} \right) \right]^{-1/\xi_i - 1} \\ &\times \frac{\Gamma(15)}{\Gamma(9)\Gamma(6)} (0.5 + \xi_i)^8 (0.5 - \xi_i)^5 \end{aligned}$$



Process Level

- Process level

$$\mu_i \sim N \left(X_i^T \beta_\mu + U_{i\mu}, \frac{1}{\tau_\mu^2} \right)$$

$$\log(\sigma_i) \sim N \left(X_i^T \beta_\sigma + U_{i\sigma}, \frac{1}{\tau_\sigma^2} \right)$$

$$\xi_i \sim N \left(X_i^T \beta_\xi + U_{i\xi}, \frac{1}{\tau_\xi^2} \right)$$

where

X_i is the **covariate** function for grid i ,

β_θ is a vector of **regression coefficients**,

τ_θ^2 is a **fixed** precision values,

$U_\theta = \{U_{1\theta}, \dots, U_{d\theta}\}$ is a **spatial random effects**

θ is generically used to stand for μ, σ , and ξ

- Random effect U_i were modelled spatially using multivariate intrinsic autoregression (IAR) model (Banerjee *et al.* 2004). IAR is a special case of conditional autoregressive (CAR) model.
- The relationship between U , U , and U is depicted through precision T matrix .

Parameter Level

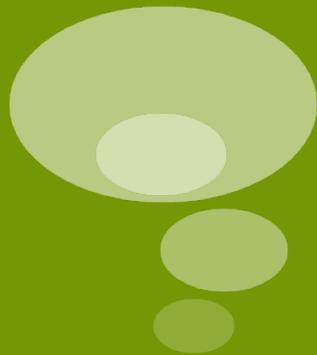
- For the two hyperparameters β and T we chose conjugate priors.
- Beta priors
$$\begin{aligned}\beta_{0\theta} &\sim N(\hat{\theta}_{mle}, 100) \\ \beta_{1\theta} &\sim N(0, 10) \\ \beta_{2\theta} &\sim N(0, 10)\end{aligned}$$
- A **Wishart** prior with 3 degree of freedom is assigned to the precision matrix T .



MCMC Implementation

- GEV parameters were drawn using a Metropolis-Hastings step with starting values are the corresponding cell-wise maximum likelihood estimates.
- Candidates for the three parameters are drawn in a block for each location j using a uniform random walk.
- Random effects U were updated following Rue & Held canonical parameterization.
- β and T were updated following its distribution.
- Codes for this work were provided by Daniel Cooley.
- The model was run for 15000 iterations for each data set, discarded the first 5000 to allow for burn-in, took only every 10th iteration results to reduce dependence.

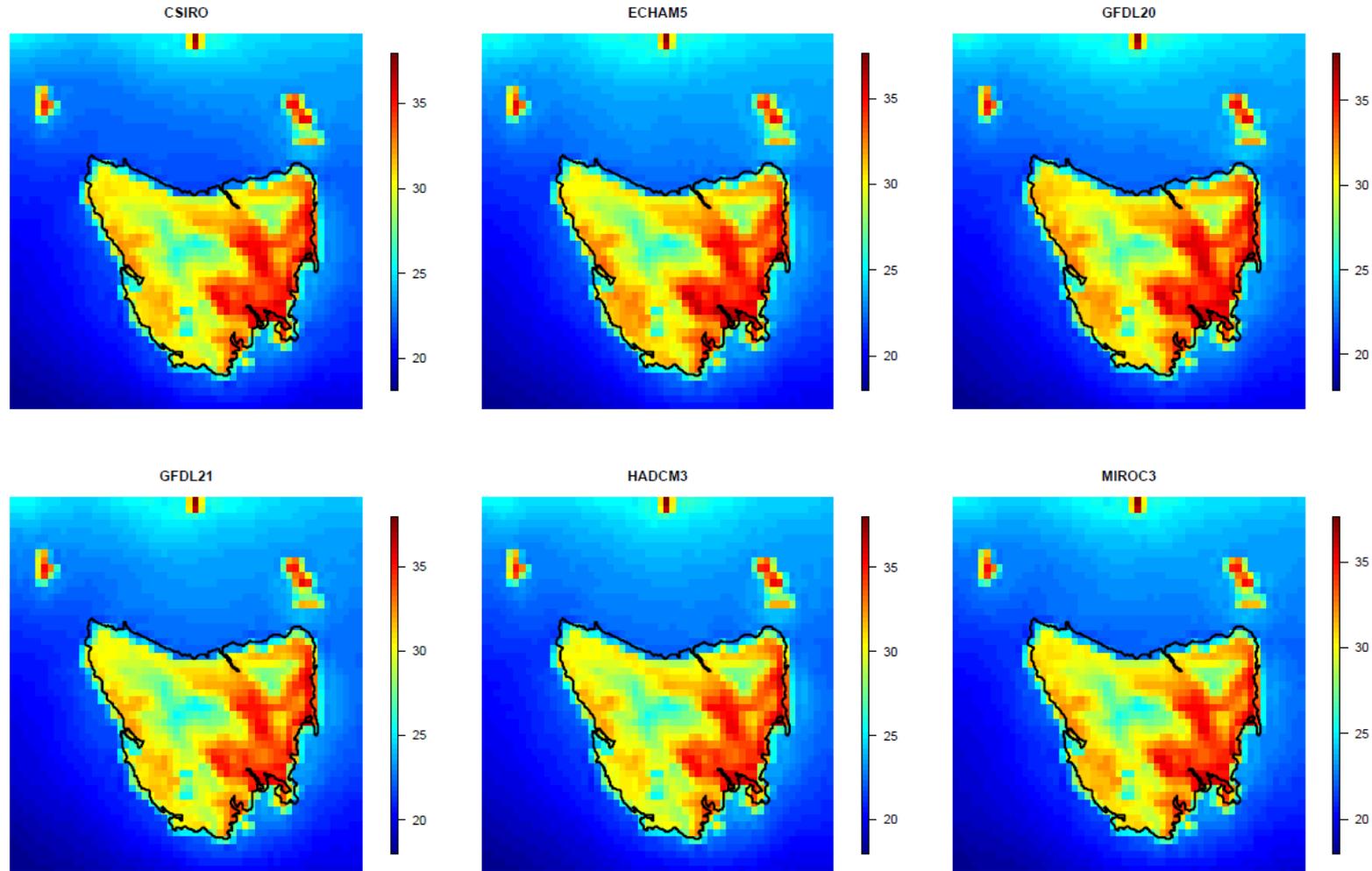




Results

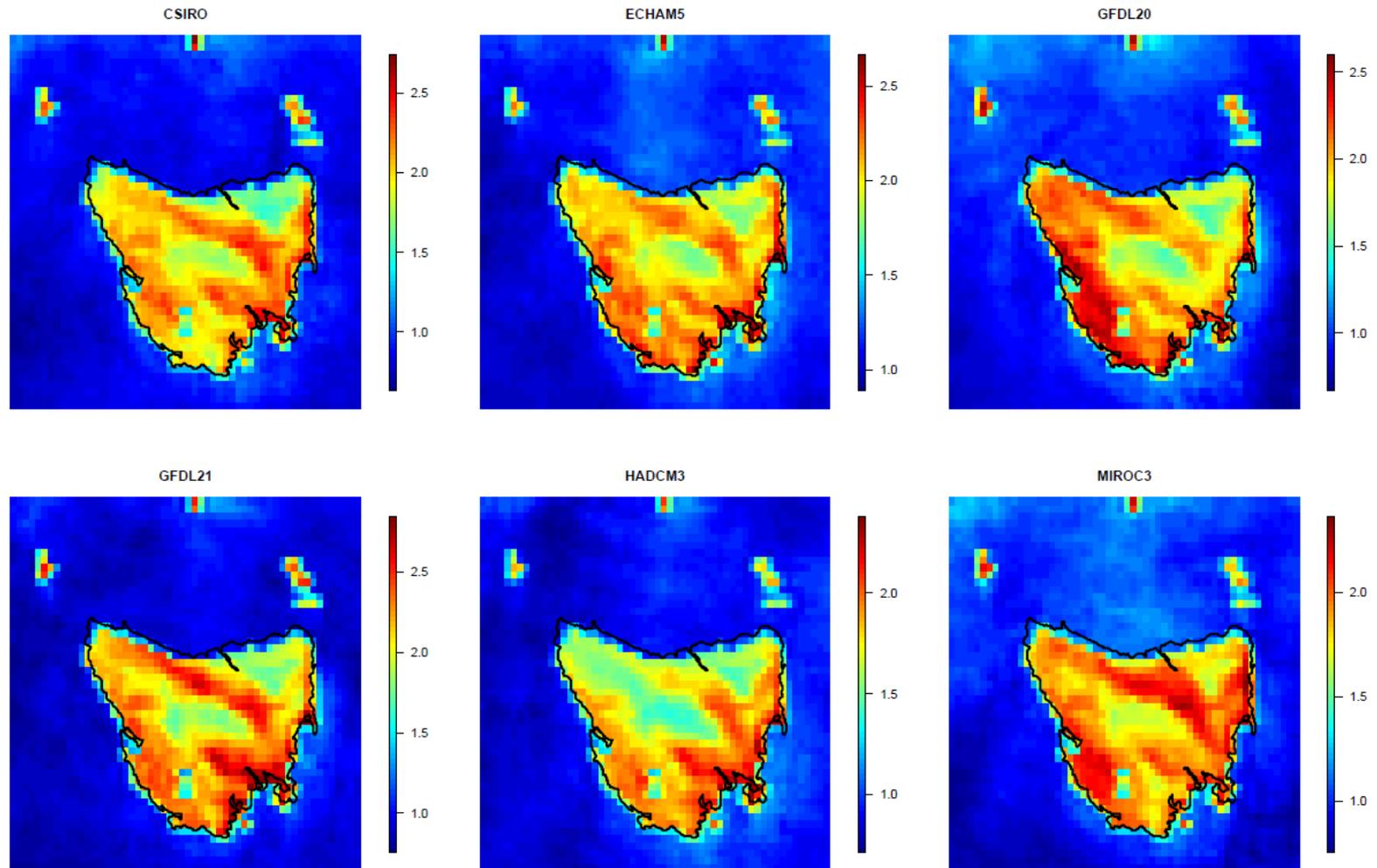
draws from posterior
distribution of μ , σ , ξ , β , U and T

Location parameters,



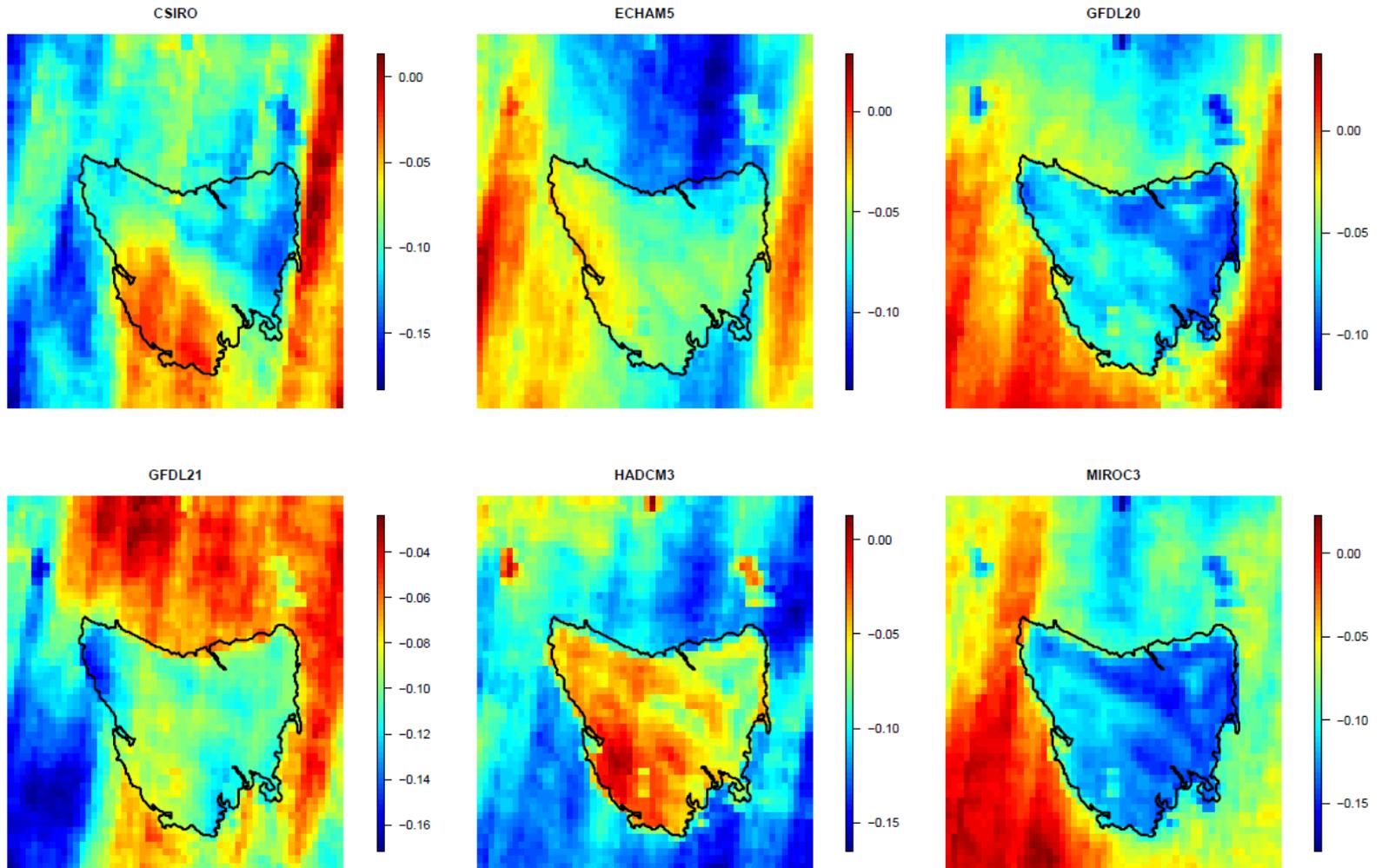
Posterior mean of **location** parameter for each simulated data driven by each of GCMs

Scale parameters,



Posterior mean of **scale** parameter for each simulated data driven by each of GCMs

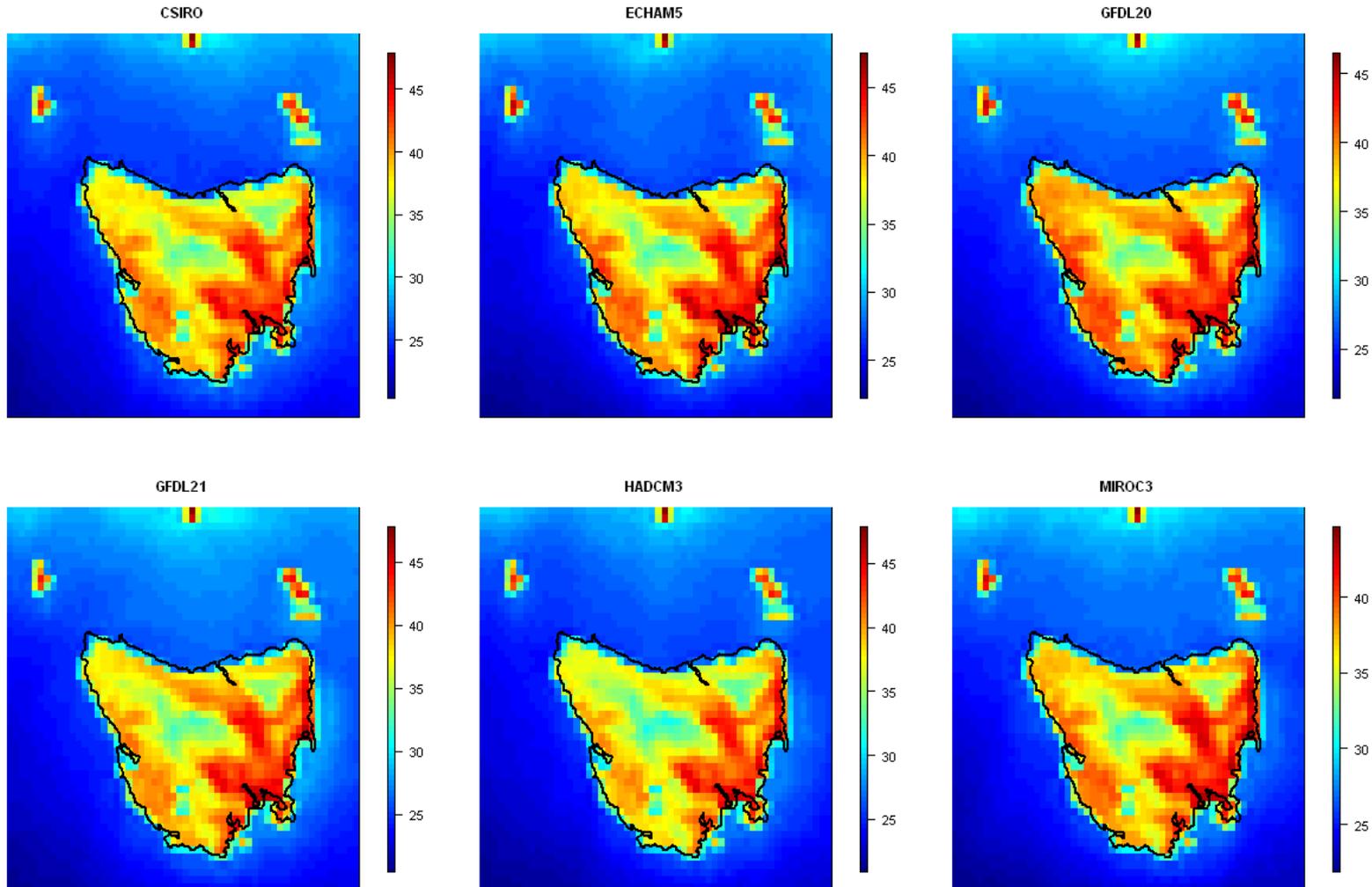
Shape parameters,



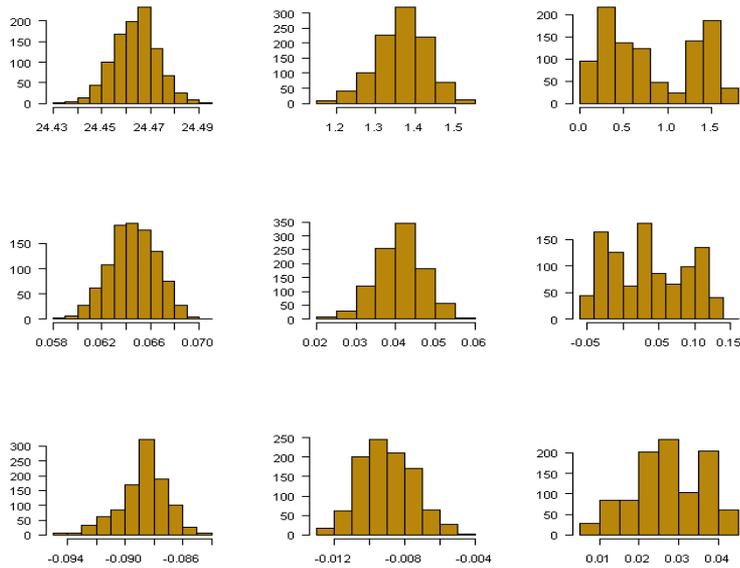
Posterior mean of **shape** parameter for each simulated data driven by each of GCMs

100 years Return levels map

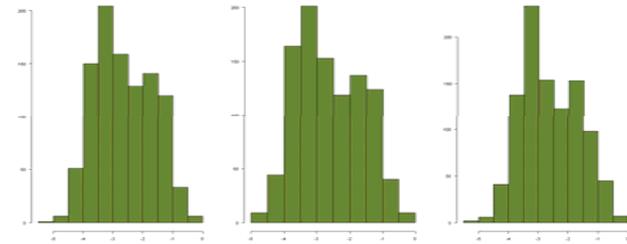
Return level basically is a **quantile** that associated with return period $1/p$.



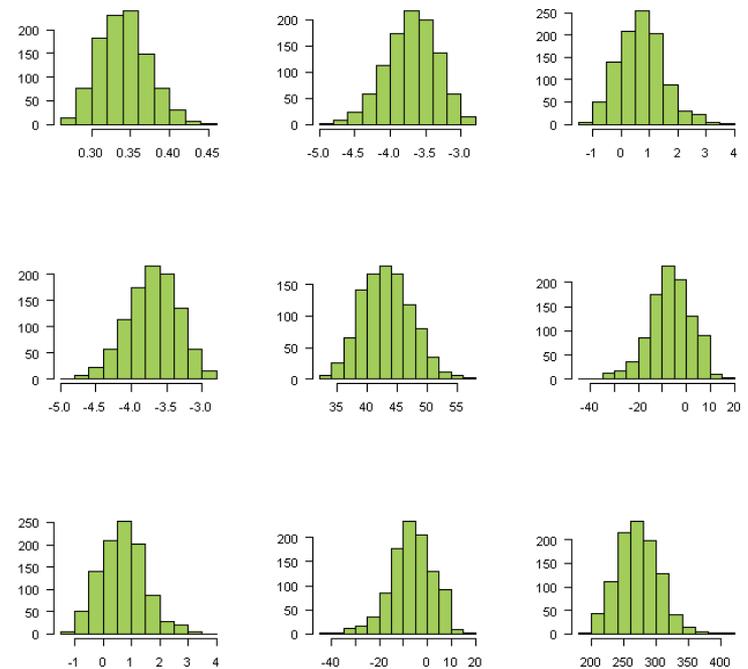
Hyperparameters Posterior



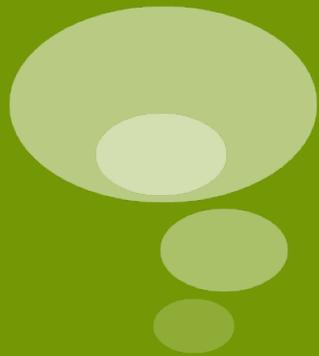
Histogram of beta posterior, the first column correspond to intercept term, second column and third correspond to latitude and longitude respectively. Each row is beta posterior for local, scale and shape parameters respectively.



Histogram of U posterior for random selected cells.



Histogram of T posterior



Conclusion

Conclusion

- We modeled maximum temperature data; simulation output from RCM driven by 6 different GCMs, using three stages Gaussian hierarchical model. The spatial patterns are not directly modeled from the data but through parameters of the assume data distribution.
- Bayesian inference was carried out by Metropolis Hastings, and following Rue & Held algorithm.
- The 100years-return level map from 6 GCMs show slightly different spatial pattern in the posterior parameter distribution especially for shape parameters.
- Model improvement can be carried on by adding covariates or considering second order spatial. Modeling observed data and comparing the model to simulation data model might enable us to find correction factor for better simulation model.
- Possible extension of this work would be to combine temperature and precipitation data using multivariate spatial hierarchical model.



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