Testing times: the effect of time period selection on empirically determined rainfall-runoff relationships

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Overview - I

Relating rainfall and runoff in a catchment enables prediction of runoff under hypothesized rainfall conditions.

Results obtained from hydrological models are very model dependent.[1,2]

An "empirical" approach infers rainfall-runoff relationships from data.

Relative changes: Suppose a quantity q takes the values
q_0 on one (baseline) time interval, q_1 on a second interval.

The relative change in q from its baseline value is

$$q' = \frac{q_1 - q_0}{q_0}. \tag{1}$$

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[1] A. Sankarasubramanian *et al.*, 'Climate elasticity of streamflow in the United States', *Water Resources Research*, **37**(6), pp. 1771-1781 (2001).

[2] J. M. Whyte *et al.* 'Comparison of predictions of rainfall-runoff models for changes in rainfall in the Murray-Darling Basin', *Hydrol. Earth Syst. Sci. Discuss.*, **8**, pp. 917-955, (2011).

Overview - II

Empirical rainfall-runoff studies are often summarized in terms such as ...

A 1% change in rainfall results in a 2-3% change in runoff for Murray–Darling Basin catchments $\longrightarrow$ example

... quite a pervasive statement in the literature on this region.

Sources of possible subjectivity in the process:

- ⋆ which test statistic is used? (E.g. monthly totals.)
- ⋆ choice of baseline period?
- ⋆ which periods are compared?

How greatly do results vary with choices made?

This talk is derived from a paper under consideration for a conference proceedings.[3]

[3] J. M. Whyte, "Estimation of precipitation elasticity of streamflow from data and variability of results" submitted to the 34th IAHR World Congress, Brisbane, June 26-July 1 2011.

An example of an empirical study

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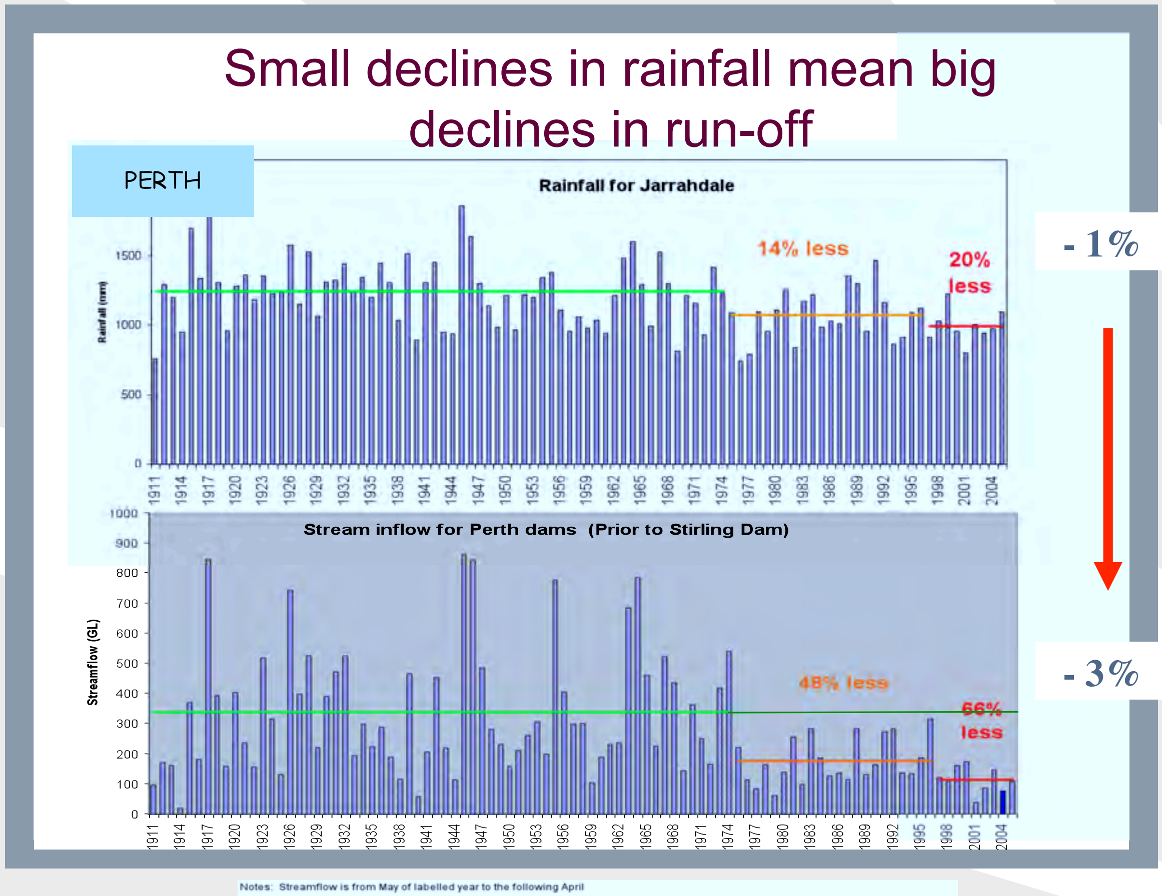

From M. Young (The Environment Institute, The University of Adelaide) “There’s a hole in the bucket Dear Liza, Dear Liza, a hole!”, Singapore 3rd Tuesday Lecture, 19th May 2009.

The model underneath the interpretation

Using notation from the literature[4]

Relative change in	runoff	from baseline value	R
	rainfall		P

are associated through

$$\frac{\delta R}{R} = \Phi \frac{\delta P}{P}, \tag{2}$$

where Φ is termed “the elasticity of runoff to change in precipitation”.

Equation (2) is a linear relationship with slope Φ passing through (0,0).

It is implicitly assumed in the interpretation of empirical study results.

[4] J. C. Schaake, ‘From Climate to Flow’, Chapter 8 in Climate Change and U.S. Water Resources, (ed. P. E. Waggoner) Wiley (1990).

Empirical association of runoff and rainfall relative changes (distilled from Whyte 2011)

Input: Catchment rainfall and runoff data (no missing values).

1. Setup:
   1. Choose a statistic of interest for rainfall and runoff.
   2. Calculate the rainfall statistic for all periods.
2. Identification of time periods of interest: Select periods for comparison and a baseline period.
3. Calculations: For each of the periods selected:
   1. Calculate the runoff statistic.
   2. Determine the relative change in the rainfall and runoff statistic.
4. Analysis of results: Infer a relationship between the change in rainfall and the apparent change in runoff statistic. (E.g. by use of (2).)

It all begins with the test statistic

The statistic of interest: a moving average over monthly totals. (Incomplete months are excluded from the data record.)

For quantity x having total x_j, $(1 \leq j \leq N)$ in month j of n_j days, moving average of window width k is

$$\bar{x}_j = \frac{x_j + \cdots + x_{j+k-1}}{n_j + \cdots + n_{j+k-1}} = \frac{\text{total of } k \text{ months of rainfall}}{\text{total days in } k \text{ month period}}, \qquad (3)$$

termed here an interval mean daily value.

When considering variables:

precipitation,	(3) gives	$\bar{p}_j$, Interval Mean Daily Precipitation (IMDaP).
runoff,		$\bar{q}_j$, Interval Mean Daily Runoff (IMDaR).

Application: Murray–Darling Basin catchments

Data from National Land and Water Resources Audit (Australia).

Catchment rainfall (mm/day) obtained by interpolation between rain gauges[5].

Considered unregulated northern Murray–Darling Basin (NSW) flow stations:

421018, Bell river at Newrea, catchment area 1620 km^2,
(data Aug 1939-June 1971, $\approx$ 32 yrs),

419010, MacDonald river at Woolbrook, catchment area 829 km^2,
(data May 1950-April 1990, $\approx$ 40 yrs)

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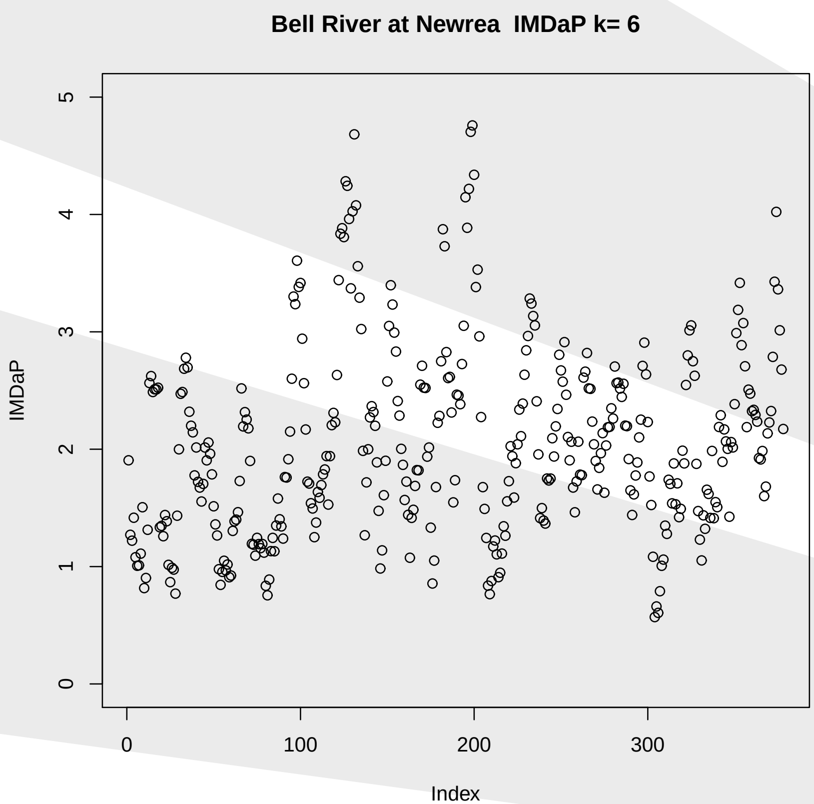
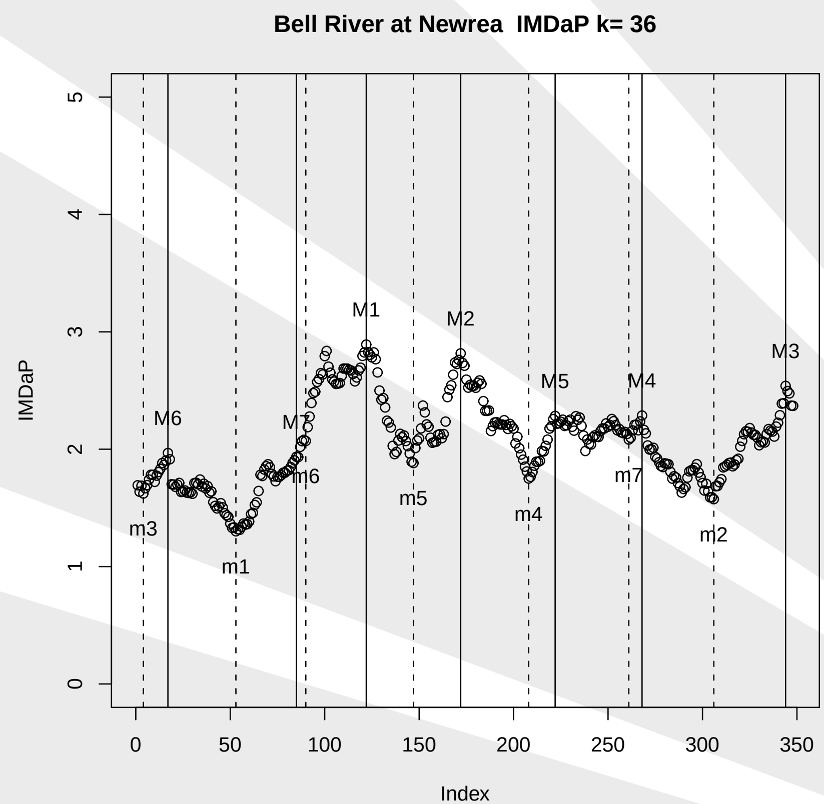
[5] S. J. Jeffrey *et al.*, "Using spatial interpolation to construct a comprehensive archive of Australian climate data", *Environmental Modelling & Software*, **16**(4), pp. 309 - 330, (2001).

Rainfall variability for one test catchment

For Bell River at Newrea catchment, consider the IMDaP values obtained for two window widths (k values)





The "window" we look through determines what we will see.

The decision points: investigation of choices

Strategies for selecting time periods:

1. Extrema selection: Place IMDaP values in increasing order, select l largest (smallest) independent values M_i (m_i) $i = 1, \ldots, l$.
2. Rainfall independent selection: take the first IMDaP produced, then take as many independent IMDaP values as the results allow.

Strategies for selecting a baseline period, IMDaP$_0$, from IMDaP selected:

1. Near median baseline: take the ceiling($N/2$)-th largest IMDaP. (This is the median value when N is odd.)
2. Maximum baseline: take the largest IMDaP. (As done in Young and McColl[6].)

[6] M. Young and J. McColl, “There’s a hole in the bucket Dear Liza, Dear Liza, a hole!”, Third Tuesday Lecture, Singapore Campus, The University of Adelaide, 19 May 2009.

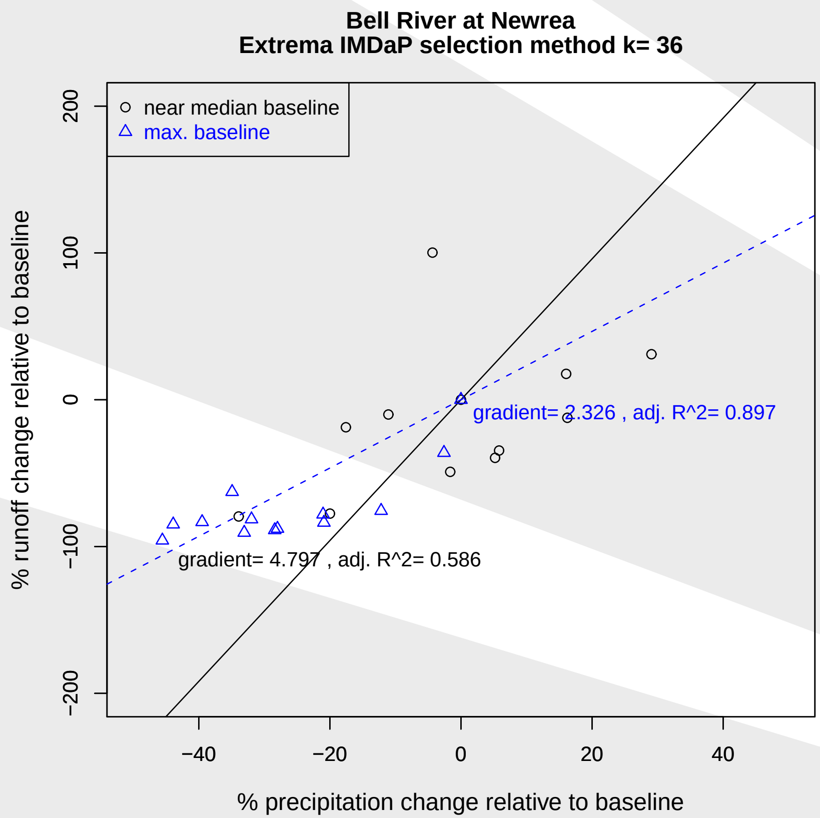
Illustration for the test catchments

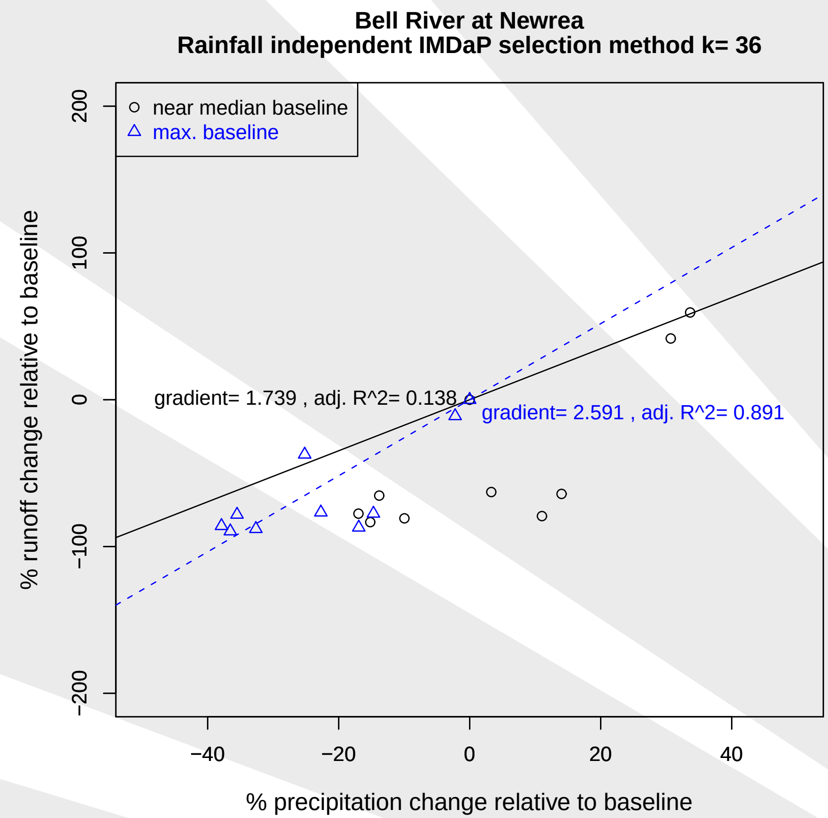
Take moving averages over rainfall for $k = 36$ months.

Apply all four combinations of selection rules for

1. periods for comparison,
2. baseline period.

Results of choices made





Lines of best fit are shown with IMDaP-IMDaR points.

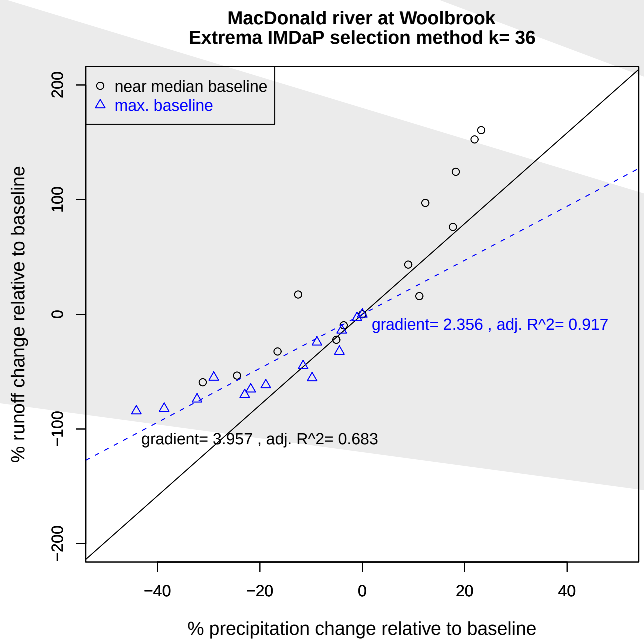
- The gradient of the line of best fit is the estimate of Φ.
  - The adjusted R^2 value indicates proportion of variability of observations explained by the model. Higher values are preferred.

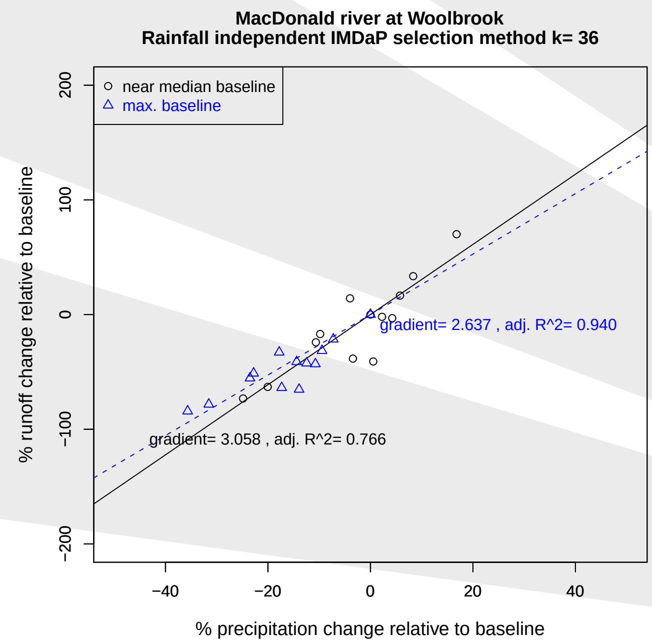
Points to note

1. If take max IMDaP as baseline then cannot comment on effect of greater rainfall values on runoff (extrapolation).

2. For certain choices Eq. (2) is not convincing. In such cases, disregard the estimate of Φ.

3. Adjusted $R^2 > 0.85$ suggests (2) fits data quite well for max. baseline method.





Results summary

All values are rounded down to two decimal places.

$\bar{R}^2$ represents adjusted R^2, $\hat{\Phi}$ is the estimate of Φ.

Catchment	**IMDaP selection method**/*baseline*			
	Extrema		**Rainfall independent**	
	near median	*max.*	*near median*	*max.*
Bell River at Newrea	$\hat{\Phi} = 4.79$ $\bar{R}^2$=0.58	$\hat{\Phi} = 2.32$ $\bar{R}^2$=0.89	$\hat{\Phi} = 1.73$ $\bar{R}^2$=0.13	$\hat{\Phi} = 2.59$ $\bar{R}^2$=0.89
MacDonald river at Woolbrook	$\hat{\Phi} = 3.95$ $\bar{R}^2$=0.68	$\hat{\Phi} = 2.35$ $\bar{R}^2$=0.91	$\hat{\Phi} = 2.63$ $\bar{R}^2$=0.94	$\hat{\Phi} = 3.05$ $\bar{R}^2$=0.76
15 Mile Creek at Greta Sth.	$\hat{\Phi} = 3.78$ $\bar{R}^2$=0.59	$\hat{\Phi} = 1.79$ $\bar{R}^2$=0.98	$\hat{\Phi} = 3.34$ $\bar{R}^2$=0.83	$\hat{\Phi} = 2.71$ $\bar{R}^2$=0.95
Ovens River at Bright	$\hat{\Phi} = 1.90$ $\bar{R}^2$=0.88	$\hat{\Phi} = 1.75$ $\bar{R}^2$=0.98	$\hat{\Phi} = 3.47$ $\bar{R}^2$=0.68	$\hat{\Phi} = 1.82$ $\bar{R}^2$=0.99

Observations and conclusions

Results for catchments considered show substantial variability of results with time periods chosen.
The range of results exceeds the $2 \leq \Phi \leq 3$ used as a rule-of-thumb for Murray-Darling catchments.

Some care is warranted when making a judgement based on one set of date periods.
It may be useful to note the baseline value when reporting results.

It is planned to consider more catchments to further test the use of the empirical method.

The systematic and and model-free method may provide a useful check on the results of hydrological models. Development is continuing.

Acknowledgements

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School of Mathematical Sciences, The University of Adelaide.

Image “Scenic view of River Murray” courtesy of the South Australian Tourism Commission.

Abstract

In an empirical study of rainfall-runoff relationships for a catchment, linear regression is commonly used to relate features of historical catchment rainfall and runoff, such as relative changes in these quantities. An example of the result of this type of study is the pervasive statement that a one percent change in rainfall is associated with approximately a two to three percent change in runoff for catchments in Australia's Murray–Darling Basin.

Applying an empirical study to a catchment data record entails choices of time periods to compare. It appears that little has been said of the sensitivity of results with choices made. It is important to gauge this sensitivity as we may expect that the value of an empirical study as a predictive tool decreases as the variability of results increases.

To explore this matter, the variability of results with time periods chosen is assessed for a selection of catchments in the northern Murray–Darling basin. Preliminary results suggest that the empirical method is very sensitive to the time periods compared as well as the time period used as the baseline period in calculating relative changes.