Some uncertainty issues in mathematical models of the environment and climate change

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Based on some of my past research and 16 years as editor-in-chief of Springer's Environmental Modeling and Assessment, I want to share with you **three observations** related to **uncertainty** and the use of **mathematical tools** to analyse **environmental problems.** These observations are:

1. Multiplicative law of probabilities can be "our friend"

2. Data based modelling can also be "our friend"

3. Cascading of errors due to model uncertainty is, in general, "our enemy".

If time allows, I will also mention a notion I call **"synchronisation of time scales"** that is needed to manage a **"sustainability screw".**

Arguably, climate change and environmental degradation are merely symptoms of the fundamental underlying problem, namely, that

Time scales of human development and natural processes are "out of synch"

Some references:

Zapert, R. Gaertner, P.S. and Filar, J.A. "Uncertainty Propagation within an Integrated Model of Climate Change", Energy Economics, Vol. 20, (1998), pp. 571-598.

Chiera, B, Filar, J.A., Zachary, D and Gordon, A., "Comparative Forecasting and a Test for Persistence in the El Nino Southern Oscillation" In: J.A. Filar and H. Haurie "Uncertainty in Environmental Decision Making", Handbook of state-of-the-art papers; Springer, New York, (2009/10) pp. 253-272.

Filar, J.A. , Krawczyk, J.B. and Agrawal, M. “Sustainability Screw” , CORE Discussion Paper 2009/28.

Boland, J., Filar, J.A. and Howlett, P.G., "Environmental Problems, Uncertainty and Mathematical Modeling", Notices of the American Math Society, vol 57 (2010), pp. 1286-1294.

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Observation 1:

- When trying to reduce risks of environmental (and other) disasters/accidents, the **multiplicative law of probabilities can be "our friend"**

- Why?

- Well, typically a sequence of (generally, dependent) events needs to occur for a disaster to occur

- For instance, A and B and C all have to occur

- Surprisingly, perhaps, it may be possible to lower $\mathbf{Pr(A \cap B \cap C)}$ by a measurable amount

- Without having a reliable estimate of $\mathbf{Pr(A \cap B \cap C)}$ itself!

How is that possible?

• Well, by the **multiplicative law of probabilities**

•**$\Pr(A \cap B \cap C) = \Pr(C|B \cap A)\ \Pr(B|A)\ \Pr(A)$**

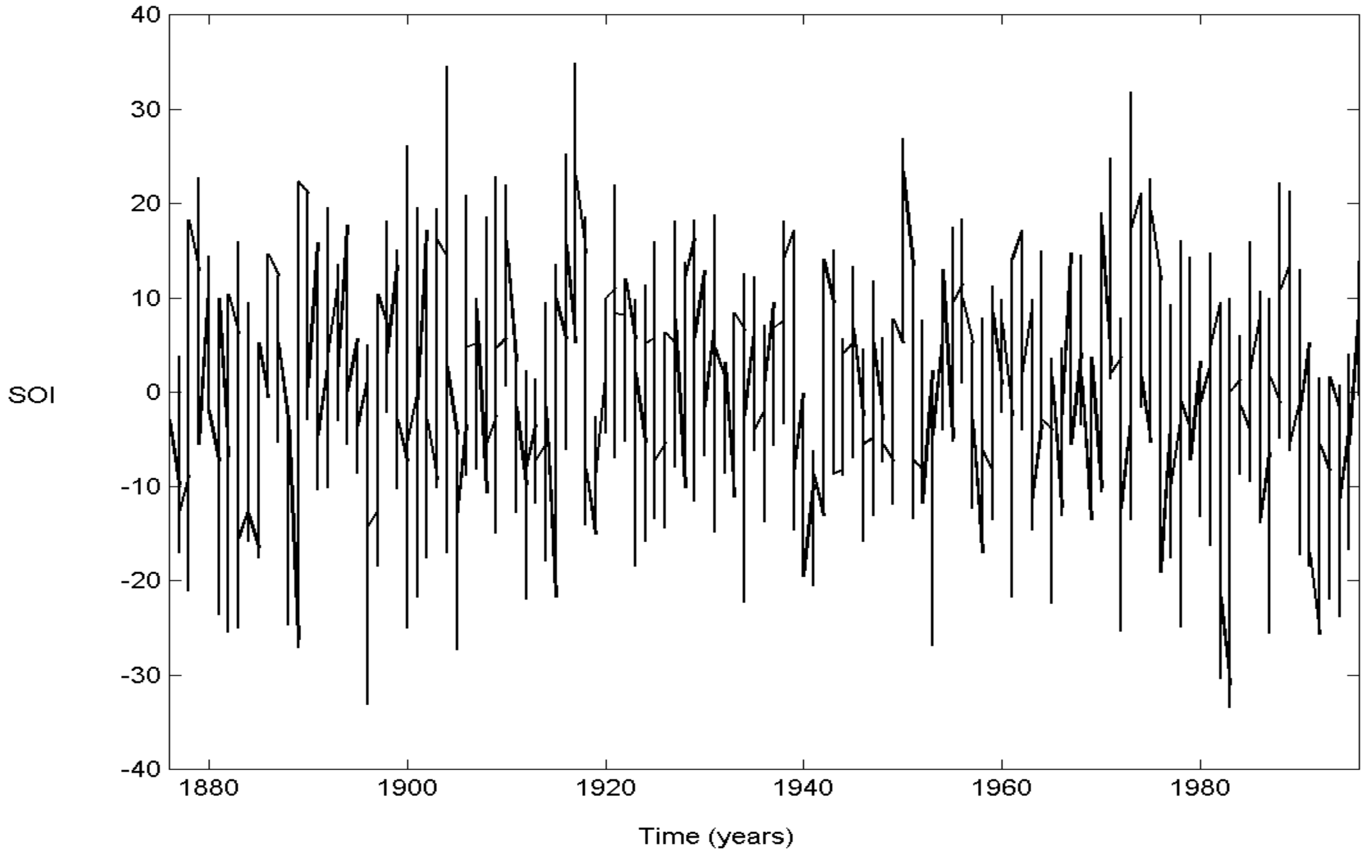
•Hence, it is sufficient to know that a certain intervention or remedial action will lower **just one** of the three factors by, say, 40%

•To claim that this particular intervention will also lower **$\Pr(A \cap B \cap C)$** by 40% without ever needing to estimate the latter probability.....

•Mathematically trivial fact but, potentially, very useful; right?

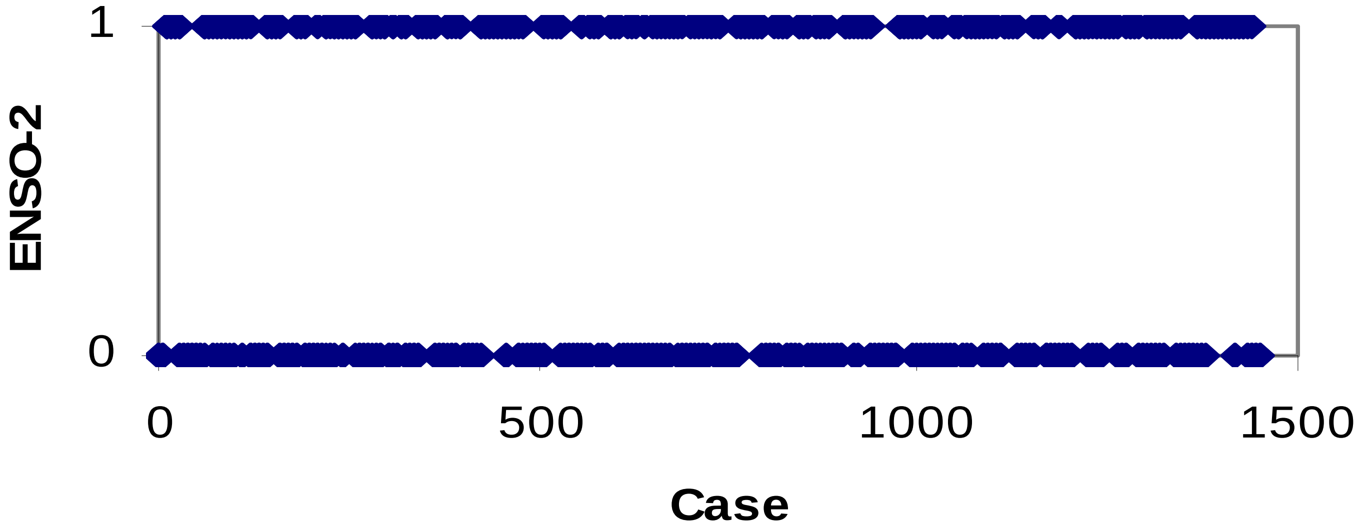
•Of course, in serious environmental risk applications, many alternative pathways to disasters need to be considered. However, fact remains that for each the probability of **just one contributing factor** needs to be lowered by a measurable amount to attain a measurable lowering of overall risk.

Observation 2: Modelling SOI Data Series



- where Troup SOI = 10 x $\dfrac{[PA(Tahiti) - PA(Darwin)] - ltavg}{SD}$

Alternative: Model an ENSO-2 Process



- Reduce the SOI series to a binary 0-1 signal where 0 represents a positive SOI value and 1 a negative SOI value.

Alternative: Model an ENSO-2 Process

- The conversion of the SOI data series to a binary 0-1 reflects the nature of the ENSO phenomenon which itself switches between an ‘ON’ and ‘OFF’ state.
- Regularities in the signal are now immediately observable suggesting an underlying level of persistence in the SOI signal.
- Model using a probability forecast.

The Bayesian ENSO-2 Model

- Define L in {0,1,2,…} as the number of 0s following a first 0 in the ENSO-2 sequence.

- N_{0i} is the number of times i 0s (i=0,...,*w*) follow a first 0.

- $N_0 = \sum_{i=0}^{w} N_{0i}$, where *w* is the largest observed sequence.

- Estimate:

$$\hat{p}_j = \begin{cases} \hat{P}(L = j) = \dfrac{N_{0j}}{N_0} & j = 0,1,\ldots,w-1 \\ \hat{P}(L \geq w) = \dfrac{N_{0w}}{N_0} & j \geq w \end{cases}$$

The Bayesian ENSO-2 Model

- Similarly, let K in {0,1,2,...} be the number of 1s following a first 1 in the ENSO-2 sequence.

- N_{1i} is the number of times i 1s (i=0,...,x) follow a first 1 in the ENSO-2 sequence.

- $N_0 = \sum_{i=0}^{x} N_{i,}$ where x is the largest observed sequence.

- Estimate:
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-

$$\hat{q}_j = \begin{cases} \hat{P}(K=j) = \dfrac{N_{1j}}{N_1} & j = 0,1,\ldots,x-1 \\ \hat{P}(K \geq x) = \dfrac{N_{1x}}{N_1} & j \geq x \end{cases}$$

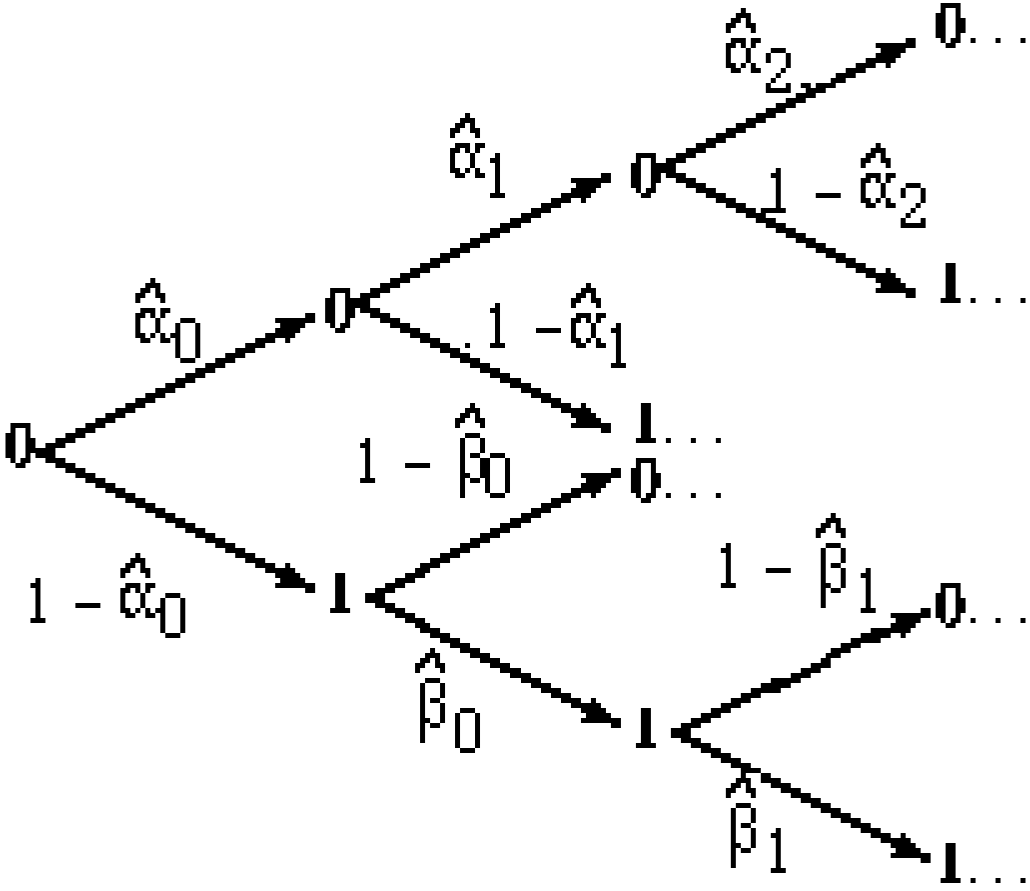
The Bayesian ENSO-2 Model

- Allowing $\overline{L}, \overline{K}$ to be the number of consecutive 0s and 1s respectively:

$$\hat{\alpha}_j = \hat{P}(\overline{L} > j+1 | \overline{L} > j) = \frac{1 - \sum_{r=1}^{j+1} \hat{P}(L = r)}{1 - \sum_{r=1}^{j} \hat{P}(L = r)}$$

$$\hat{\beta}_j = \hat{P}(\overline{K} > j+1 | \overline{K} > j) = \frac{1 - \sum_{r=1}^{j+1} \hat{P}(K = r)}{1 - \sum_{r=1}^{j} \hat{P}(K = r)}$$

The Bayesian ENSO-2 Model



Converting ENSO-2 to SOI

1. Consider a 0-1 sequence as being composed of *blocks* of 0s and 1s.

$$\underbrace{1111}\ \underbrace{0}\ \underbrace{111}\underbrace{000}\ \underbrace{1}\ \ \underbrace{0}\ \ \underbrace{1}\ \underbrace{0000000000}$$

2. On the basis of the original SOI series, construct tables of average SOI values for runs of lengths 1,2,3,… and replace each block of 0s and 1s with the corresponding SOI run.

3. Simple & performs rather well for forecasts of 1-6 months.

Observation 3: Cascading of errors due to model uncertainty is, in general, “our enemy”

The **nature** and the **magnitude** of the **errors** that accompany a model's outputs/forecasts must be understood.

These can be classified into a number of distinct types; including the (initially small?) modelling errors: ***Em.***

•A big problem with the latter is that they may **cascade**!
•This is especially true for models whose solutions are iterated over many time steps (like climate models).

• A related (fascinating) issue is whether some climate parameters should really be seen as slow variables and whether human activities are steering them towards critical values where chaos kicks in. I will not talk about that.

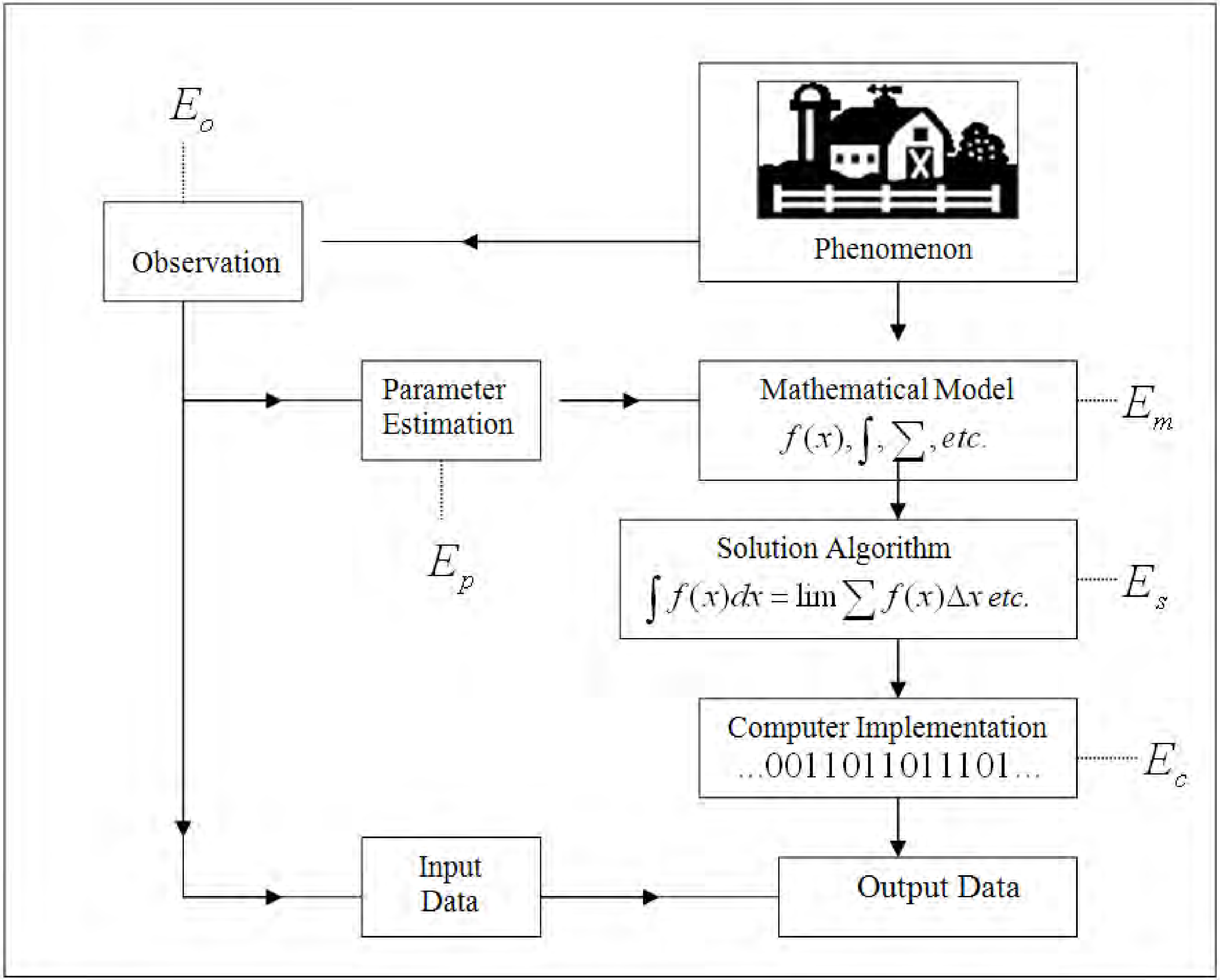
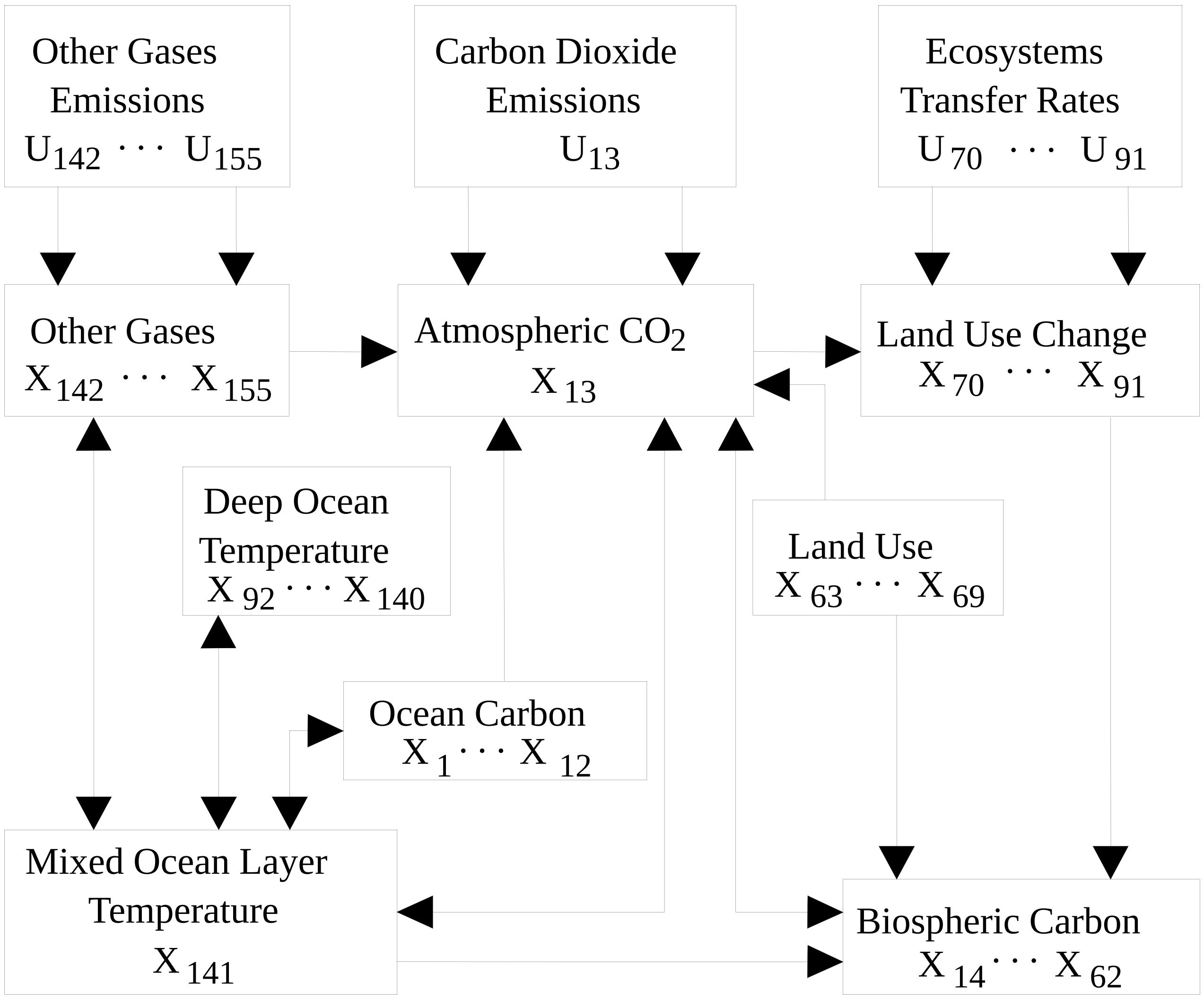


Figure 2. Sources of uncertainty in the outputs of mathematical models.

Climate Dynamics of IMAGE 1.0 (cont.)



Climate Dynamics of IMAGE 1.0

The essential dynamic interactions of IMAGE 1.0 can be represented as:

$$\frac{d\,X(t)}{dt} = A\,X(t) + N(X(t)) + U(t)$$

$$= F(X(t)) + U(t) \qquad X_0 = X(1990)$$

where

$$A = (A_{IJ}), \quad I, J = x, y, z, s, S, u, v, w$$

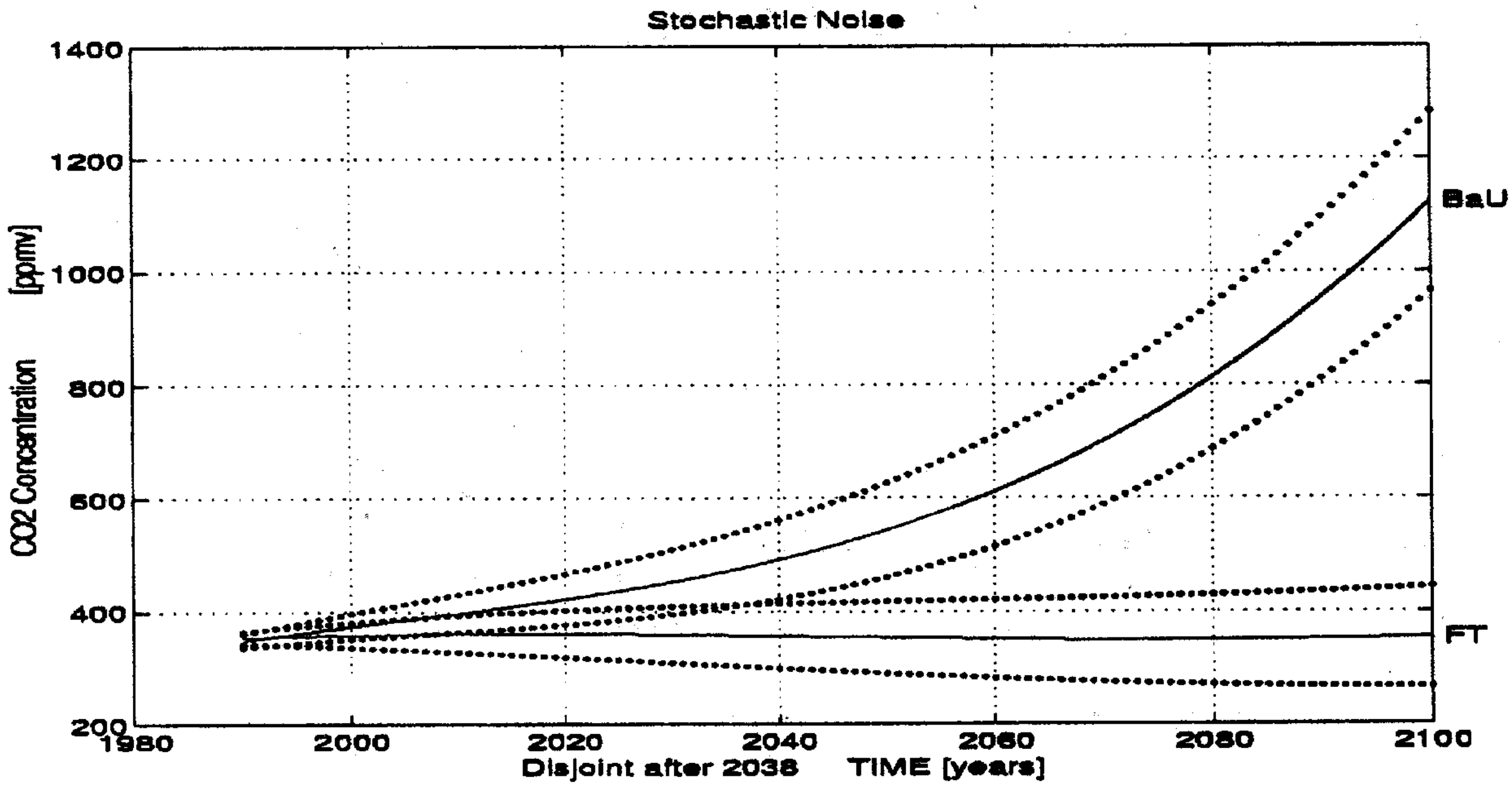
What if ***Em*** came in the form of a small amount of white noise?

$$dX(t) = [F(X(t)) + U(t)]\,dt + G\,dW(t); \qquad X_0 = C(1990)$$

a systems of stochastic differential equations.

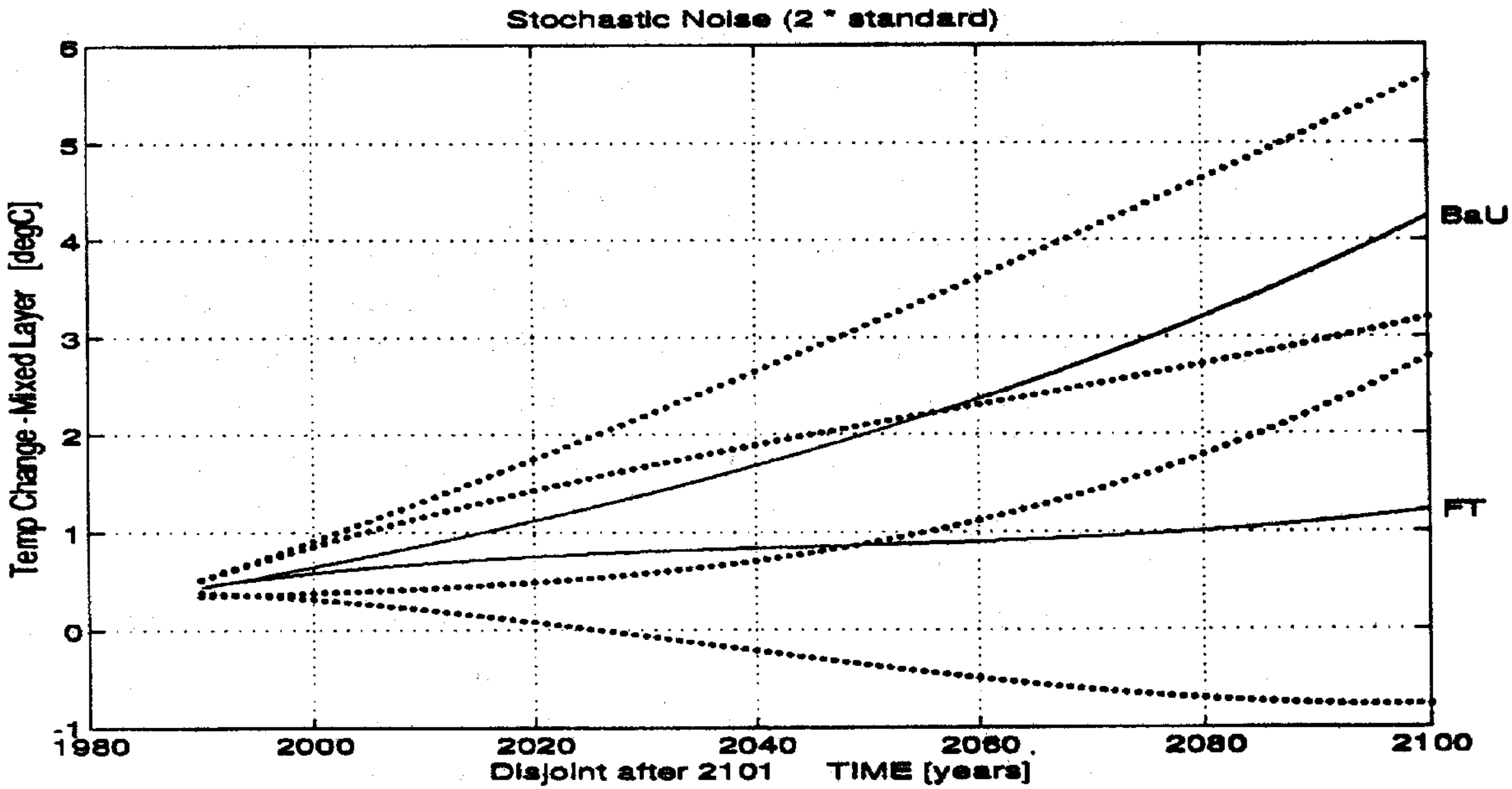
Scenario Overlap

Slides 1 and 2 depict simulations for the two standard scenarios BaU and FT, produced under two different conditions of uncertainty.



Overlap of CO2 conc predictions for the BaU and FT scenarios.
Uncertainty induced by stochastic noise.

Scenario Overlap (cont.)



Overlap of the temperature change of the mixed ocean layer predictions for the BaU and FT scenarios with deterministic initial conditions and stochastic noise double magnitude.

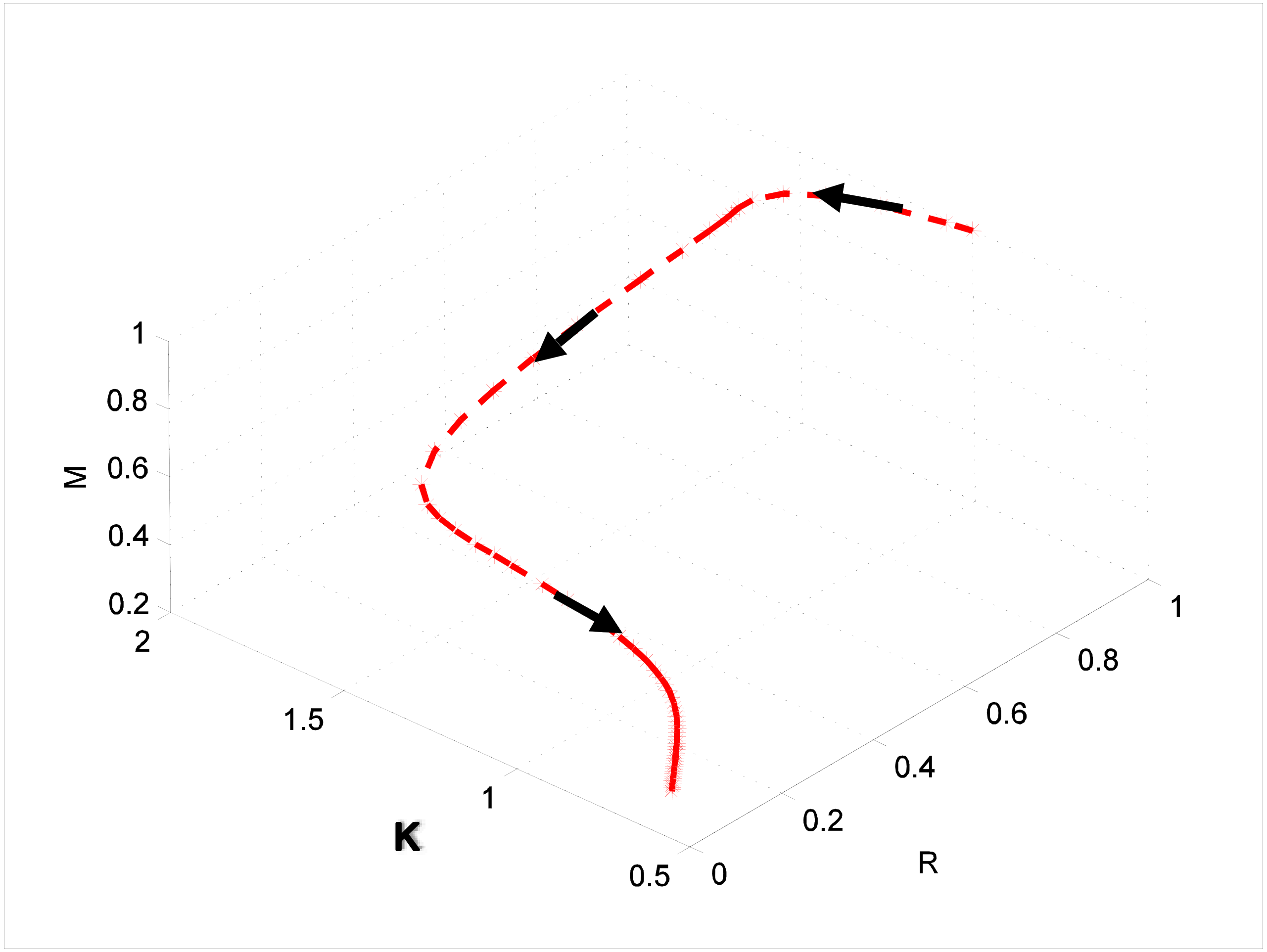
So, while cascading ***Em*** is our enemy

- Detecting whether, where and how strongly it features, can be of great help
- Stochastic differential equations are just one of many possible tools (e.g., statistics, Monte Carlo, importance sampling, conditional-value-at-risk...) suitable for this kind of ``detective work"
- Ultimately, it would be desirable to have a comprehensive error analysis framework that – for a given model – attributes the "correct portion" of possible error to
- ***Eo, Ep, Es, Ec and Em***
- The fact that most environmental (especially climate) models involve processes with very disparate time scales contributes to the importance of understanding ***Em*** better.

Sustainability Screw

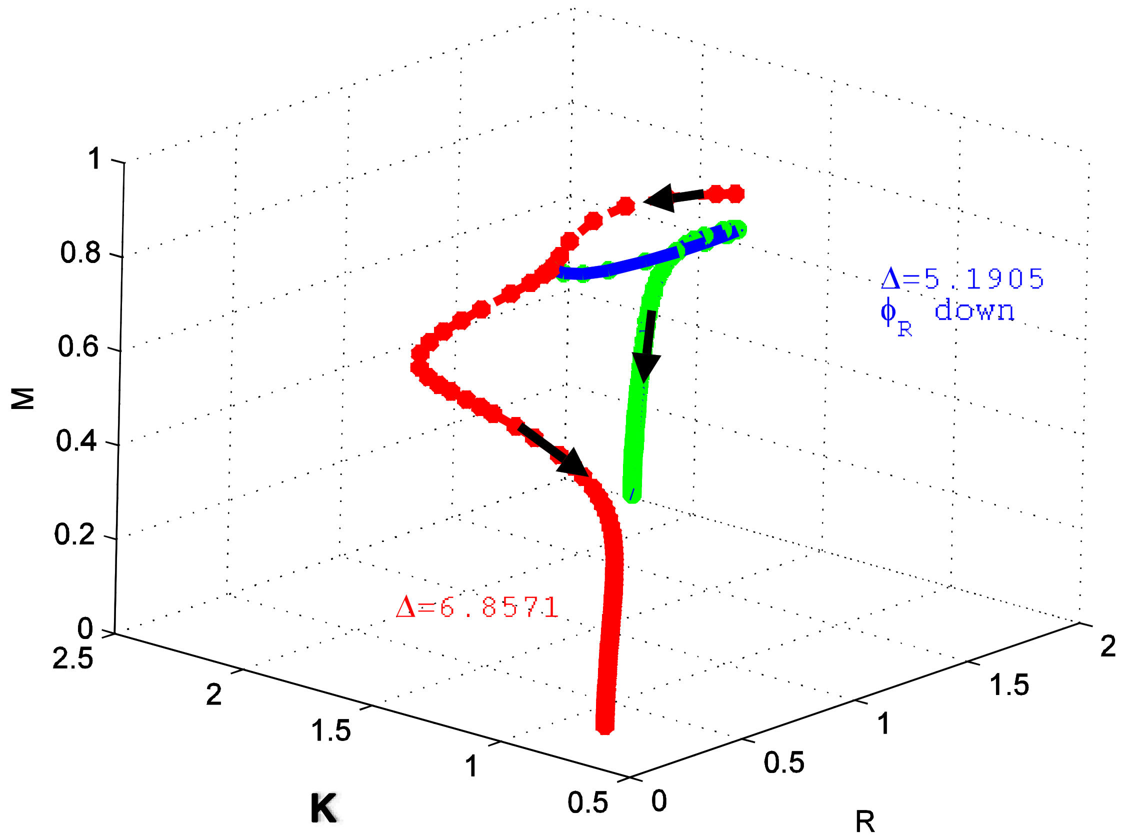
- Recently, we constructed an artificially simple dynamical system just to try to illustrate the importance of varying time scales.
- The nature-man system that we proposed is portrayed by only five, coupled, ordinary differential equations.
- Non-renewable resource
- Renewable resource
- Production capital
- Abatement capital
- Human capital

The default velocities (time scales) of these equations were calibrated to imitate global surrogates such as oil reserves, biocapacity, population, etc.....



Sustainability screw

If BaU is pursued, then all three: the non-renewable resources M, the renewable resources R and the production capital K are spiralling down to catastrophic (?) levels.



Bifurcated sustainability screws

Green is the averted screw; while the **red** is with business as usual

Importance of time scales

To drive this point home consider a “trick” in the analysis of systems with fast and slow motions:

“Stretch time” by setting: $$\tau = \frac{t}{\varepsilon}$$

Set: $$\varepsilon = 1/31{,}250{,}000$$ that is,

1 hour = 31,250,000 years, 1 day = 750,000,000 years

and 6 days = 4.5 billion years = history of the earth

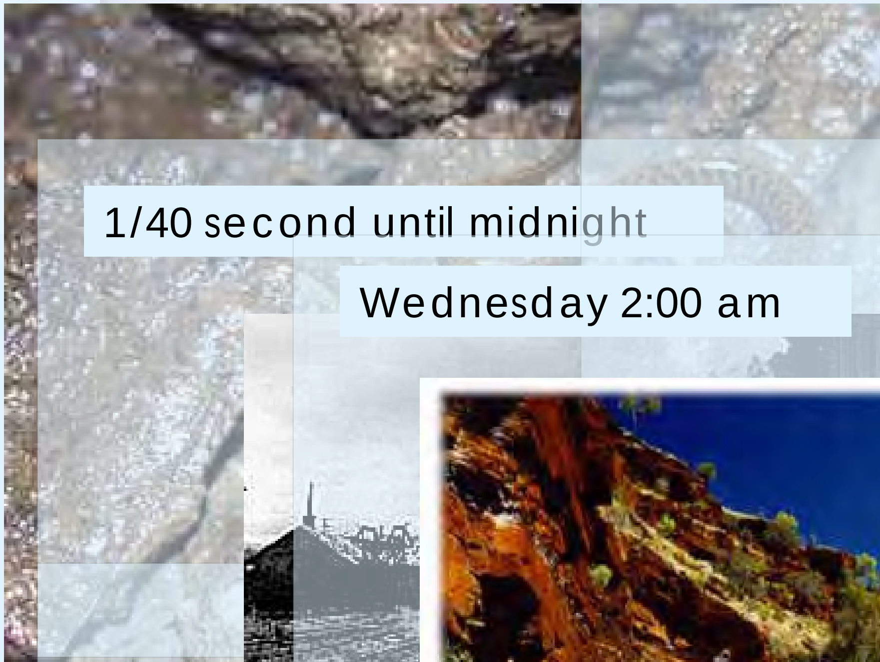
This was done by David Brower.

Saturday 4:00 pm – 9:55 pm

Saturday 11:59:44

Monday 12:00 am

Saturday morning



1/40 second until midnight

Wednesday 2:00 am

n of the earth

Bacteria

Industrial revolution

Photosynthesis fully deve

Fortescue Falls - Pilbara

lgae

I’ll return to an earlier claim:

Climate change and environmental degradation are merely symptoms of the fundamental underlying problem, namely, that :

Time scales of human development and natural processes are “out of synch”

Synchronization of these time scales is essential to any meaningful mitigation and adaptation strategies. Otherwise, they will be “Band-Aid” remedies, at best.

“It’s the time, stupid!”

Questions?

- Fairly good forecast results for up to six months and a breakdown of predictability power thereafter after.
- A lot of literature, however, points to the same conclusion for schemes that are much more complicated.
- For example, sophisticated methods including modeling of oscillations via equatorial wave dynamics in coupled ocean-atmosphere systems have shown comparable results.
- Schopf, P. S., Suarez, M.J.: Vacillations in a coupled ocean-atmosphere model. J. Atm. Sci. 45, pp.549-566 (1988)
- Stammer, D., Wunsch, C., Giering, R., Eckert, C., Heimbach, P., Marotzke, J., Adcroft, A., Hill, C.N., Marshall, J.: The global ocean circulation during 1992-1997, estimated from ocean observations and a general circulation model. J. Geophys. Res., 107 (C9) (2002)